Metastable periodic Fréedericksz transition in nematic liquid crystals

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The stability of twist periodic deformations induced by a magnetic field on a nematic liquid crystal sample planarly oriented is analyzed. We show that this kind of deformation always corresponds to a metastable state. This conclusion is in agreement with the experimental observation, according to which this type of distortion is not stable in time. [S1063-651X(96)06111-9]

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Let us consider a nematic sample planarly oriented along the x axis. A distorting magnetic field is applied along the y axis, perpendicular to the initial nematic orientation. The two surfaces of the sample are at $z=\pm d/2$, where d is the thickness of the slab. In this situation the deformation induced by the magnetic field can be a pure twist characterized by the twist angle Φ formed by the nematic director $\vec{n}$ with the x axis. As is well known [1], this reorienting phenomenon, in the case of strong anchoring on the two limiting surfaces, has a well defined threshold given by

$$H_c=\frac{\pi}{d}\sqrt{\frac{K_{22}}{\chi_a}}, \tag{1}$$

where K is the twist elastic constant, and $\chi_a=\chi_\parallel-\chi_\perp$ the diamagnetic anisotropy ($\parallel$ and $\perp$ refer to $|\vec{n}$). This kind of Fréederiksz transition is usually called ''aperiodic'' and $\Phi=\Phi(z)$ only. A few years ago Lonberg and Meyer have shown that the elastic anisotropy can induce periodic distortions instead of aperiodic ones [2]. They have also shown that when these periodic distortions appear, they involve spatial deformations. Hence the distortion does not correspond to a simple twist deformation. This means that the nematic director $\vec{n}$ is not everywhere parallel to a plane. The critical values for the elastic anisotropies necessary to observe this kind of periodic Fréedericksz transition for different surface treatments and geometries have been discussed by several authors [3–6]. More recently [7] it has been shown, both experimentally and theoretically, that a periodic deformation completely described by the angle Φ in the planar geometry described above can appear. This kind of Fréedericksz transition, that from now on we shall call twist periodic instability, has been theoretically analyzed by Evangelista and coworkers [8,9]. In this Brief Report we want to show that this twist periodic deformation is metastable, in the sense that its energy is always larger than that of the aperiodical one.

Let $\vec{n}(x,z)=\vec{i}\cos\Phi(x,z)+\vec{j}\sin\Phi(x,z)$ the nematic liquid crystal (NLC) director for a deformation of the kind discussed above. In the situation under consideration, the free energy density f of the NLC is [8]

$$f=\frac{1}{2}\left\{K_{11}\sin^2\Phi\left(\frac{\partial\Phi}{\partial x}\right)^2+K_{22}\left(\frac{\partial\Phi}{\partial z}\right)^2+K_{33}\cos^2\Phi\left(\frac{\partial\Phi}{\partial x}\right)^2-\chi_aH^2\sin^2\Phi\right\}, \tag{2}$$

where K_{11}, K_{22}, and K_{33} are the splay, twist, and bend elastic constants, respectively [1]. In the limit of small Φ the previous expression for f reduces to

$$f=\frac{1}{2}\left\{K_{33}\left(\frac{\partial\Phi}{\partial x}\right)^2+K_{22}\left(\frac{\partial\Phi}{\partial z}\right)^2-\chi_aH^2\Phi^2\right\}. \tag{3}$$

For the aperiodic Fréedericksz transition $\Phi=\Phi_{ap}(z)$. We suppose strong anchoring conditions on the two surfaces, which implies $\Phi(-d/2)=\Phi(d/2)=0$. In this framework, at the lowest order, we can assume

$$\Phi_{ap}(z)=A_{ap}\cos(\pi z/d), \tag{4}$$

which is the first term of the expansion of $\Phi_{ap}(z)$ in Fourier's series in $(-d/2,d/2)$. By substituting Eq. (4) into Eq. (3) and integrating over z, from $-d/2$ to $d/2$, and x, from 0 to λ, simple calculations give for the free energy per unit length the expression

$$F_{ap}=\int_{-d/2}^{d/2}f_{ap}dz=\frac{\pi^2K_{22}\lambda}{4d}\left\{1-\frac{\chi_aH^2}{K_{22}}\left(\frac{d}{\pi}\right)^2\right\}A_{ap}^2. \tag{5}$$

From Eq. (5) one deduces that the undistorted state, corresponding to $A_{ap}=0$, is stable for

$$H<H_{ap}=\frac{\pi}{d}\sqrt{\frac{K_{22}}{\chi_a}}, \tag{6}$$

i.e., for fields lower than the critical field given by Eq. (1).

Let us consider now the twist periodic Fréedericksz transition for which, at the lowest order, Φ can be assumed to be of the form

$$\Phi=\Phi_p(x,z)=A_p\cos(2\pi x/\lambda)\cos(\pi z/d). \tag{7}$$

By substituting Eq. (7) into Eq. (3) and integrating over z, from $-d/2$ to $d/2$, and x, over one period, simple calculations give

$$F_p=\int_{-d/2}^{d/2}\int_0^{\lambda} f_p dx\ dz=\frac{\pi^2 K_{22}\lambda}{8d}\left\{\left(2\frac{d}{\lambda}\right)^2\frac{K_{33}}{K_{22}}+1-\frac{\chi_a H^2}{K_{22}}\left(\frac{d}{\pi}\right)^2\right\}A_p^2. \tag{8}$$

From Eq. (8) we obtain that the undistorted state, corresponding to $A_p=0$, is stable for

$$H<H_p(\lambda)=\frac{\pi}{d}\sqrt{\frac{K_{22}}{\chi_a}\left[1+\left(2\frac{d}{\lambda}\right)^2\frac{K_{33}}{K_{22}}\right]}>H_{ap}. \tag{9}$$

Since $H_p(\lambda)>H_{ap}$, $\forall\lambda$, it follows that the stable state, distorted under the action of the magnetic field, is always the aperiodic one. Hence the periodic distortion of the kind given by Eq. (7) could represent a metastable state.

A simple analysis shows that $F_p>F_{ap}$ for $H\geqslant H_c$. Let us suppose Φ of the form $\Phi_p(x,z)=[1+\epsilon(x)]\Phi_{ap}(z)$, where $\epsilon(x)$ is a periodic function. This form for $\Phi_p(x,z)$ follows from the boundary conditions $\Phi(x,\pm d/2)=0$ due to the strong anchoring hypothesis. Simple calculations give for the total free energy per unit length of the periodic distortion, near the aperiodic one (i.e., for $\epsilon\to 0$),

$$F_p=\int_{-d/2}^{d/2}\int_0^{\lambda} f_p dx\ dz=\int_{-d/2}^{d/2}\int_0^{\lambda}\frac{1}{2}K_{33}\Phi_{ap}^2(z)\left(\frac{d\epsilon}{dx}\right)^2 dx\ dz+F_{ap}\frac{1}{\lambda}\int_0^{\lambda}[1+\epsilon^2(x)]dx>F_{ap}, \tag{10}$$

where

$$F_{ap}=\int_{-d/2}^{d/2}\int_0^{\lambda}\frac{1}{2}\left[K_{22}\left(\frac{d\Phi_{ap}}{dz}\right)^2-\chi_a H^2\Phi_{ap}^2\right]dx\ dz. \tag{11}$$

Equation (10) shows that in the limit $\Phi_{ap}\ll 1$, and $\epsilon\ll 1$, even if Eq. (4) and Eq. (7) do not hold, $F_p>F_{ap}$. In other words, for $H>H_c$ the stable state is the aperiodic one. Hence a periodic twist deformation induced by a magnetic field in a planarly oriented nematic sample is a metastable state. For this reason the periodic patterns observed in [7] disappear after some time.

In our analysis we have only considered planar arrangement of the director by a static point of view. Srajer *et al.* [10] have shown that spatially periodic structures correspond to nonequilibrium configurations. Their model distinguishes two regimes: the first is dominated by the viscosities, and the second one by the elasticity. This simply means that the total energy of the spatially periodic deformations is higher than the one-dimensional solution for the distortion.

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